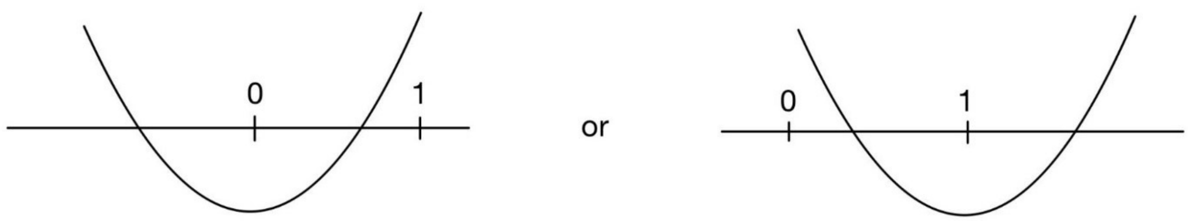
Q) The set of all real values of λ for which the quadratic equations, $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval (0, 1) is :

Solution:

Given quadratic equation, $(\lambda^2+1)x^2-4\lambda x+2=0$
Here coefficient of x^2 is $(\lambda^2 + 1)$ which is always positive. So quadratic equation is upward parabola.



So, either $f(0) < 0$ and $f(1) > 1$
or $f(0) > 0$ and $f(1) < 0$
$\therefore$ In both those cases, $f(0)\ f(1) \le 0$
$\Rightarrow 2(\lambda^2 - 4\lambda + 3) \le 0$
$\Rightarrow \lambda \in [1, 3]$
At $\lambda = 1$:
Quadratic equation becomes $2x^2 - 4x + 2 = 0$
$\Rightarrow (x - 1)^2 = 0$
$\Rightarrow x = 1, 1$
As both roots can't lie between (0, 1)
So, $\lambda = 1$ can't be possible.
At $\lambda = 3$: $10x^2 - 12x + 2 = 0$
$\Rightarrow 5x^2 - 6x + 1 = 0$
$\Rightarrow (5x - 1)(x - 1) = 0$
$\Rightarrow x = 1, 1/5$
In the interval (0, 1) exactly one root 1/5 present.

$\therefore \lambda \in (1, 3]$