Q) If a + b + c = 1, ab + bc + ca = 2 and abc = 3, then the value of $a^4 + b^4 + c^4$ is equal to _____.

Solution:

$(a + b + c)^2 = 1$

$\Rightarrow a^2 + b^2 + c^2 + 2(ab + bc + ca) = 1$

$\Rightarrow a^2 + b^2 + c^2 = -3$ ....(i)

$\Rightarrow ab + bc + ca = 2$ ....(ii)

Squaring of equation (ii),

$\Rightarrow a^2b^2 + b^2c^2 + c^2a^2 + 2(ab^2c + bc^2a + ca^2b) = 4$

$\Rightarrow a^2b^2 + b^2c^2 + c^2a^2 + 2abc(a + b + c) = 4$

$\Rightarrow a^2b^2 + b^2c^2 + c^2a^2 + 6 = 4$

$\Rightarrow a^2b^2 + b^2c^2 + c^2a^2 = -2$ ....(iii)

Squaring of equation (i),

$\Rightarrow a^4 + b^4 + c^4 + 2(a^2b^2 + b^2c^2 + c^2a^2) = 9$

$\Rightarrow a^4 + b^4 + c^4 - 4 = 9$

$\Rightarrow a^4 + b^4 + c^4 = 13$