

Q) If $a = \cos(2\pi/7) + i \sin(2\pi/7)$, then the quadratic equation whose roots are $\alpha = a + a^2 + a^4$ and $\beta = a^3 + a^5 + a^6$ is _____.

Solution:

$$a = \cos(2\pi/7) + i \sin(2\pi/7)$$

$$a^7 = [\cos(2\pi/7) + i \sin(2\pi/7)]^7$$

$$= \cos 2\pi + i \sin 2\pi = 1 \dots (i)$$

$$S = \alpha + \beta = (a + a^2 + a^4) + (a^3 + a^5 + a^6)$$

$$S = a + a^2 + a^3 + a^4 + a^5 + a^6$$

$$= \frac{a(1-a^6)}{1-a}$$

$$\text{or } S = (a - 1)/(1 - a)$$

$$= -1 \dots (ii)$$

$$P = \alpha * \beta = (a + a^2 + a^4) (a^3 + a^5 + a^6)$$

$$= a^4 + a^6 + a^7 + a^5 + a^7 + a^8 + a^7 + a^9 + a^{10}$$

$$= a^4 + a^6 + 1 + a^5 + 1 + a + 1 + a^2 + a^3 \text{ (From eqn (i))}$$

$$= 3 + (a + a^2 + a^3 + a^4 + a^5 + a^6)$$

$$= 3 + S = 3 - 1 = 2 \text{ [From (ii)]}$$

Required equation is, $x^2 - Sx + P = 0$

$$x^2 + x + 2 = 0.$$