

Q) The number of solutions of the equation $\log_{x=1} 2x^2 + 7x + 5 + \log_{(2x+5)}(x + 1)^2 - 4 = 0$, $x > 0$, is

Correct Answer is 1

Explanation

$$\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x + 1)^2 - 4 = 0$$

$$\log_{(x+1)}(2x + 5)(x + 1) + 2 \log_{(2x+5)}(x + 1) = 4$$

$$\log_{(x+1)}(2x + 5) + 1 + 2 \log_{(2x+5)}(x + 1) = 4$$

$$\text{Put } \log_{(x+1)}(2x + 5) = t$$

$$t + 2/t = 3 \Rightarrow t^2 - 3t + 2 = 0$$

$$t = 1, 2$$

$$\log_{(x+1)}(2x + 5) = 1 \text{ \& } \log_{(x+1)}(2x + 5) = 2$$

$$x+1=2x+3 \text{ \& } 2x+5=(x+1)^2$$

$$x=-4 \text{ (rejected)}$$

$$x^2=4 \Rightarrow x=2, -2 \text{ (rejected)}$$

$$\text{So, } x = 2$$

No. of solution = 1