Q) Given that the equation $z^2 + (p + iq)z + r + is = 0$, where p, q, r, s are real and non-zero has a real root, then how are p, q, r and s related?

Solution:

Given that $z^2 + (p + iq)z + r + is = 0$ ......(i)

Let z = α (where α is real) be a root of (i), then

$α^2 + (p + iq)α + r + is = 0$ or

$α^2 + pα + r + i(qα + s) = 0$

Equating real and imaginary parts, we have $α^2 + pα + r = 0$ and $qα + s = 0$

Eliminating α, we get

$(-s / q)^2 + p(-s / q) + r = 0$ or

$s^2 - pqs + q^2r = 0$ or

$pqs = s^2 + q^2r$