Q) The difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and $a \neq b$, then what is the relation between a and b?

Solution:

Let α, β and γ,δ be the roots of the equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$, respectively therefore, $\alpha + \beta = -a$, $\alpha\beta = b$ and $\delta + \gamma = -b$, $\gamma\delta = a$.

Given $|\alpha - \beta| = |\gamma - \delta| \Rightarrow (\alpha + \beta)^2 - \alpha\beta$

$= (\gamma + \delta)^2 - 4\gamma\delta$

$\Rightarrow a^2 - 4b = b^2 - 4a$

$\Rightarrow (a^2 - b^2) + 4(a - b) = 0$

$\Rightarrow a + b + 4 = 0$ (Because $a \neq b$)