

Q) If $b_1 b_2 = 2 (c_1 + c_2)$, then at least one of the equations $x^2 + b_1 x + c_1 = 0$ and $x^2 + b_2 x + c_2 = 0$ has _____ roots.

Solution:

Let D_1 and D_2 be discriminants of $x^2 + b_1 x + c_1 = 0$ and $x^2 + b_2 x + c_2 = 0$, respectively.

Then,

$$D_1 + D_2 = b_1^2 - 4c_1 + b_2^2 - 4c_2$$

$$= (b_1^2 + b_2^2) - 4(c_1 + c_2)$$

$$= b_1^2 + b_2^2 - 2b_1 b_2 \text{ [Because } b_1 b_2 = 2(c_1 + c_2)\text{]} = (b_1 - b_2)^2 \geq 0$$

$\Rightarrow D_1 \geq 0$ or $D_2 \geq 0$ or D_1 and D_2 both are positive.

Hence, at least one of the equations has real roots.