

Q) The common roots of the equations $x^{12} - 1 = 0$, $x^4 + x^2 + 1 = 0$ are _____.

Solution:

$$x^{12} - 1 = (x^6 + 1)(x^6 - 1)$$

$$= (x^6 + 1)(x^2 - 1)(x^4 + x^2 + 1)$$

Common roots are given by $x^4 + x^2 + 1 = 0$

$$x^2 = \frac{-1 \pm i\sqrt{3}}{2} = \omega, \omega^2 \text{ or } \omega^4, \omega^2 \text{ (Because } \omega^3 = 1) \text{ or}$$

$$x = \pm \omega^2, \pm \omega$$