

If a, b, c are real and $x^3 - 3b^2x + 2c^3$ is divisible by $x - a$ and $x - b$, then

A) $a = -b = -c$

B) $a = 2b = 2c$

C) $a = b = c, a = -2b = -2c$

D) None of these

Correct Answer: C

Solution:

As $f(x) = x^3 - 3b^2x + 2c^3$ is divisible by $x - a$ and $x - b$, therefore

$$f(a) = 0 \Rightarrow a^3 - 3b^2a + 2c^3 = 0 \dots\dots(i) \text{ and}$$

$$f(b) = 0 \Rightarrow b^3 - 3b^3 + 2c^3 = 0 \dots\dots(ii)$$

From (ii), $b = c$

$$\text{From (i), } a^3 - 3ab^2 + 2b^3 = 0$$

(Putting $b = c$)

$$\Rightarrow (a - b) (a^2 + ab - 2b^2) = 0$$

$$\Rightarrow a = b \text{ or } a^2 + ab = 2b^2$$

Thus $a = b = c$ or $a^2 + ab = 2b^2$ and $b = c, a^2 + ab = 2b^2$

is satisfied by $a = -2b$.

But $b = c$.

$a^2 + ab - 2b^2 = 0$ and $b = c$ is equivalent to $a = -2b = -2c$