3. The sum $\sum_{k=1}^{20} k\frac{1}{2^k}$ is equal to

[JEE Main 2019, 8 April Shift-II]

(a) $2-\frac{11}{2^{19}}$ (b) $1-\frac{11}{2^{20}}$

(c) $2-\frac{3}{2^{17}}$ (d) $2-\frac{21}{2^{20}}$

Exp. **(a)**

Let $S=\sum_{k=1}^{20} k\left(\frac{1}{2^k}\right)$

$$S=\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\frac{4}{2^4}+\ldots+\frac{20}{2^{20}} \quad \ldots(i)$$

On multiplying by $\left(\frac{1}{2}\right)$ both sides, we get

$$\frac{S}{2}=\frac{1}{2^2}+\frac{2}{2^3}+\frac{3}{2^4}+\ldots+\frac{19}{2^{20}}+\frac{20}{2^{21}} \quad \ldots(ii)$$

On subtracting Eq. (ii) from Eq. (i), we get

$$S-\frac{S}{2}=\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots+\frac{1}{2^{20}}-\frac{20}{2^{21}}$$

$$\Rightarrow \quad \frac{S}{2}=\frac{\frac{1}{2}\left(1-\frac{1}{2^{20}}\right)}{1-\frac{1}{2}}-\frac{20}{2^{21}}$$

$$\left[\because \text{sum of GP}=\frac{a(1-r^n)}{1-r}, r<1\right]$$

$$=1-\frac{1}{2^{20}}-\frac{20}{2^{21}}=1-\frac{1}{2^{20}}-\frac{10}{2^{20}}=1-\frac{11}{2^{20}}$$

$$\Rightarrow \quad S=2-\frac{11}{2^{19}}$$