1. The sum of the series

$$2\cdot{}^{20}C_0+5\cdot{}^{20}C_1+8\cdot{}^{20}C_2+11\cdot{}^{20}C_3+\ldots+62\cdot{}^{20}C_{20}$$ is equal to

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(a) 2^{26} (b) 2^{25}

(c) 2^{23} (d) 2^{24}

Exp. (b)

Given series is

$$2\cdot{}^{20}C_0+5\cdot{}^{20}C_1+8\cdot{}^{20}C_2+\ldots+62\cdot{}^{20}C_{20}$$

$$=\sum_{r=0}^{20}(3r+2)\cdot{}^{20}C_r$$

[∵ general term of the sequence 2, 5, 8, …, which forms an AP, is $2+(n-1)3=3n-1$, where $n=1,2,3\ldots$ and it can be written as $3n+2$, where $n=0,1,2,3$]

$$=3\cdot\sum_{r=0}^{20}r\,{}^{20}C_r+2\sum_{r=0}^{20}{}^{20}C_r$$

$$=3\sum_{r=1}^{20}r\left(\frac{20}{r}\right){}^{19}C_{r-1}+2\sum_{r=0}^{20}{}^{20}C_r$$

$$\left[\because {}^nC_r=\frac{n}{r}\,{}^{n-1}C_{r-1}\right]$$

$$=3\times20\sum_{r=1}^{20}{}^{19}C_{r-1}+2\sum_{r=0}^{20}{}^{20}C_r$$

$$=60\sum_{r=0}^{19}{}^{19}C_r+2\sum_{r=0}^{20}{}^{20}C_r$$

$$\left[\because \sum_{r=1}^{20}{}^{19}C_{r-1}=\sum_{r=0}^{19}{}^{19}C_r\right]$$

$$=(60\times2^{19})+(2\times2^{20})\qquad\left[\because \sum_{r=0}^{n}{}^nC_r=2^n\right]$$

$$=(15\times2^{21})+2^{21}$$

$$=16\times2^{21}=2^{25}$$