

**Example 10** If  $a_1, a_2, \dots, a_n$  are in A.P. with common difference  $d$  (where  $d \neq 0$ ); then the sum of the series  $\sin d (\operatorname{cosec} a_1 \operatorname{cosec} a_2 + \operatorname{cosec} a_2 \operatorname{cosec} a_3 + \dots + \operatorname{cosec} a_{n-1} \operatorname{cosec} a_n)$  is equal to  $\cot a_1 - \cot a_n$

**Solution** We have

$$\begin{aligned} & \sin d (\operatorname{cosec} a_1 \operatorname{cosec} a_2 + \operatorname{cosec} a_2 \operatorname{cosec} a_3 + \dots + \operatorname{cosec} a_{n-1} \operatorname{cosec} a_n) \\ &= \sin d \left[ \frac{1}{\sin a_1 \sin a_2} + \frac{1}{\sin a_2 \sin a_3} + \dots + \frac{1}{\sin a_{n-1} \sin a_n} \right] \\ &= \frac{\sin(a_2 - a_1)}{\sin a_1 \sin a_2} + \frac{\sin(a_3 - a_2)}{\sin a_2 \sin a_3} + \dots + \frac{\sin(a_n - a_{n-1})}{\sin a_{n-1} \sin a_n} \\ &= \frac{\sin a_2 \cos a_1 - \cos a_2 \sin a_1}{\sin a_1 \sin a_2} + \frac{\sin a_3 \cos a_2 - \cos a_3 \sin a_2}{\sin a_2 \sin a_3} + \dots + \frac{\sin a_n \cos a_{n-1} - \cos a_n \sin a_{n-1}}{\sin a_{n-1} \sin a_n} \\ &= (\cot a_1 - \cot a_2) + (\cot a_2 - \cot a_3) + \dots + (\cot a_{n-1} - \cot a_n) \\ &= \cot a_1 - \cot a_n \end{aligned}$$