

Types of Relations

- **Empty Relation**

An empty relation (or void relation) is one in which there is no relation between any elements of a set. For example, if set $A = \{1, 2, 3\}$ then, one of the void relations can be $R = \{x, y\}$ where, $|x - y| = 8$. For empty relation,

$$R = \varnothing \subset A \times A$$

- **Universal Relation**

A universal (or full relation) is a type of relation in which every element of a set is related to each other. Consider set $A = \{a, b, c\}$. Now one of the universal relations will be $R = \{x, y\}$ where, $|x - y| \geq 0$. For universal relation,

$$R = A \times A$$

- **Identity Relation**

In an identity relation, every element of a set is related to itself only. For example, in a set $A = \{a, b, c\}$, the identity relation will be $I = \{a, a\}, \{b, b\}, \{c, c\}$. For identity relation,

$$I = \{(a, a), a \in A\}$$

- **Inverse Relation**

Inverse relation is seen when a set has elements which are inverse pairs of another set. For example, if set $A = \{(a, b), (c, d)\}$, then inverse relation will be $R^{-1} = \{(b, a), (d, c)\}$. So, for an inverse relation,

$$R^{-1} = \{(b, a): (a, b) \in R\}$$

- **Reflexive Relation**

In a reflexive relation, every element maps to itself. For example, consider a set $A = \{1, 2\}$. Now an example of reflexive relation will be $R = \{(1, 1), (2, 2), (1, 2), (2, 1)\}$. The reflexive relation is given by-

$$(a, a) \in R$$

- **Symmetric Relation**

In a symmetric relation, if $a=b$ is true then $b=a$ is also true. In other words, a relation R is symmetric only if $(b, a) \in R$ is true when $(a, b) \in R$. An example of symmetric relation will be $R = \{(1, 2), (2, 1)\}$ for a set $A = \{1, 2\}$. So, for a symmetric relation,

$$aRb \Rightarrow bRa, \forall a, b \in A$$

- **Transitive Relation**

For transitive relation, if $(x, y) \in R, (y, z) \in R$, then $(x, z) \in R$. For a transitive relation,

$$aRb \text{ and } bRc \Rightarrow aRc \forall a, b, c \in A$$

- **Equivalence Relation**

If a relation is reflexive, symmetric and transitive at the same time it is known as an equivalence relation.