

Check whether the relation  $R$  in  $\mathbb{R}$  defined as  $R = \{(a, b) : a \leq b^3\}$  is reflexive, symmetric or transitive.

**Solution:**

$$R = \{(a, b) : a \leq b^3\}$$

$$\left(\frac{1}{2}, \frac{1}{2}\right) \notin R, \text{ since } \frac{1}{2} > \left(\frac{1}{2}\right)^3$$

$\therefore R$  is not reflexive.

$$(1, 2) \in R \text{ (as } 1 < 2^3 = 8)$$

$$(2, 1) \notin R \text{ (as } 2^3 > 1 = 8)$$

$\therefore R$  is not symmetric.

$$\left(3, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{6}{5}\right) \in R, \text{ since } 3 < \left(\frac{3}{2}\right)^3 \text{ and } \frac{2}{3} < \left(\frac{6}{5}\right)^3$$

$$\left(3, \frac{6}{5}\right) \notin R \text{ as } 3 > \left(\frac{6}{5}\right)^3$$

$\therefore R$  is not transitive.

$R$  is neither reflexive nor symmetric nor transitive.