

Show that the relation R in \mathbb{R} defined as $R = \{(a,b) : a \leq b\}$ is reflexive and transitive, but not symmetric.

Solution:

$$R = \{(a,b) : a \leq b\}$$

$$(a,a) \in R$$

$\therefore R$ is reflexive.

$$(2,4) \in R \text{ (as } 2 < 4)$$

$$(4,2) \notin R \text{ (as } 4 > 2)$$

$\therefore R$ is not symmetric.

$$(a,b), (b,c) \in R$$

$$a \leq b \text{ and } b \leq c$$

$$\Rightarrow a \leq c$$

$$\Rightarrow (a,c) \in R$$

$\therefore R$ is transitive.

R is reflexive and transitive but not symmetric.