

Show that the relation R in the set \mathbb{R} of real numbers, defined as $R = \{(a, b) : a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

Solution:

$$R = \{(a, b) : a \leq b^2\}$$

$$\left(\frac{1}{2}, \frac{1}{2}\right) \notin R \quad \text{because } \frac{1}{2} > \left(\frac{1}{2}\right)^2$$

$\therefore R$ is not reflexive.

$(1, 4) \in R$ as $1 < 4$. But 4 is not less than 1^2 .

$$(4, 1) \notin R$$

$\therefore R$ is not symmetric.

$$(3, 2)(2, 1.5) \in R \quad [\text{Because } 3 < 2^2=4 \text{ and } 2 < (1.5)^2=2.25]$$

$$3 > (1.5)^2 = 2.25$$

$$\therefore (3, 1.5) \notin R$$

$\therefore R$ is not transitive.

R is neither reflexive nor symmetric nor transitive.