27 Consider a circular current-carrying loop of radius R in the x-yplane with centre at origin. Consider the line integral

$$\Im(L) = \left| \int_{-L}^{L} \mathbf{B.dl} \right|$$

taken along z-axis.

(a) Show that $\Im(L)$ monotonically increases with L

(b) Use an appropriate amperian loop to show that $\Im(\infty) = \mu_0 I$. where I is the current in the wire

(c) Verify directly the above result

(d) Suppose we replace the circular coil by a square coil of sides R carrying the same current I.

What can you say about $\Im(L)$ and $\Im(\infty)$?

K **Thinking Process**

This question revolves around the application of Ampere circuital law.

(a) B (z) points in the same direction on z-axis and hence, $J(L)$ is a monotonically function of L.

Since, B and **dl** along the same direction, therefore **B**. **dl** = **B**. **dl** as cos 0 = 1

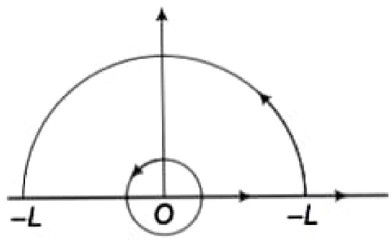
(b) $J(L)$ + contribution from large distance on contour $C = \mu_0 I$

∴ as $L \to \infty$

Contribution from large distance → 0 (as $B \propto 1/r^3$)

$$J(\infty) - \mu_0 I$$

(c) The magnetic field due to circular current-carrying loop of radius R in the x-y plane with centre at origin at any point lying at a distance of from origin.



$$B_z = \frac{\mu_0 I R^2}{2(z^2 + R^2)^{3/2}}$$

$$\int_{-\infty}^{\infty} B_z dz = \int_{-\infty}^{\infty} \frac{\mu_0 I R^2}{2(z^2 + R^2)^{3/2}} dz$$

Put $z = R \tan \theta_1$

⇒ $dz = R \sec^2 \theta d\theta$

∴ $$\int_{-\infty}^{\infty} B_z dz = \frac{\mu_0 I}{2} \int_{-\pi/2}^{\pi/2} \cos\theta d\theta = \mu_0 I$$

(d) $B(z)_{square} < B(z)_{circular\ coil}$

∴ $$\Im(L)_{square} < \Im(L)_{circular\ coil}$$

But by using arguments as in (b)

$$\Im(\infty)_{square} = \Im(\infty)_{circular}$$