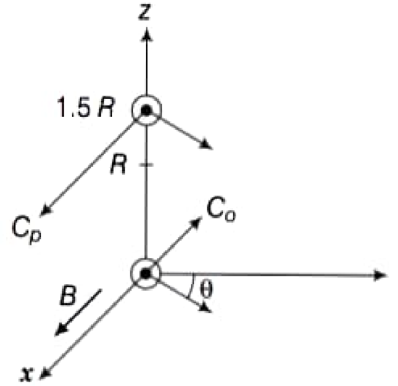
25 An electron and a positron are released from (0, 0, 0) and (0, 0, 1.5R) respectively, in a uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{i}}$, each with an equal momentum of magnitude $p = eBR$. Under what conditions on the direction of momentum will the orbits be non-intersecting circles?

K **Thinking Process**

The circles of the electron and a positron shall not overlap if the distance between the two centers are greater than 2R

Since, B is along the x-axis, for a circular orbit the momenta of the two particles are in the y-z plane. Let p_1 and p_2 be the momentum of the electron and positron, respectively. Both traverse a circle of radius R of opposite sense. Let p_1 make an angle θ with the y-axis p_2 must make the same angle.



The centres of the respective circles must be perpendicular to the momenta and at a distance R. Let the centre of the electron be at C_e and of the positron at C_p. The coordinates of C_e is

$$C_e \equiv (0, -R\sin\theta, R\cos\theta)$$

The coordinates of C_p is

$$C_p \equiv (0, -R\sin\theta, \frac{3}{2}R - R\cos\theta)$$

The circles of the two shall not overlap if the distance between the two centers are greater than 2R.

Let d be the distance between C_p and C_e.

Let d be the distance between C_p and C_e.

Then,
$$d^2 = (2R\sin\theta)^2 + \left(\frac{3}{2}R - 2R\cos\theta\right)^2$$

$$= 4R^2\sin^2\theta + \frac{9^2}{4}R - 6R^2\cos\theta + 4R^2\cos^2\theta$$

$$= 4R^2 + \frac{9}{4}R^2 - 6R^2\cos\theta$$

Since, d has to be greater than 2R

$$d^2 > 4R^2$$

$\Rightarrow$
$$4R^2 + \frac{9}{4}R^2 - 6R^2\cos\theta > 4R^2$$

$\Rightarrow$
$$\frac{9}{4} > 6\cos\theta$$

or,
$$\cos\theta < \frac{3}{8}$$