Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that
(i) the problem is solved
(ii) exactly one of them solves the problem.

Solution:

Probability of solving the problem by A, $P(A)=\frac{1}{2}$

Probability of solving the problem by B, $P(B)=\frac{1}{3}$

Since the problem is solved independently by A and B,

$$P(AB)=P(A)\times P(B)=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$$

$$P(A')=1-P(A)=1-\frac{1}{2}=\frac{1}{2}$$

$$P(B')=1-P(B)=1-\frac{1}{3}=\frac{2}{3}$$

(i) Probability that the problem is solved $=P(A\cup B)$

$$=P(A)+P(B)-P(AB)$$

$$=\frac{1}{2}+\frac{1}{3}-\frac{1}{6}$$

$$=\frac{4}{6}=\frac{2}{3}$$

(ii) Probability that exactly one of them solves the problem $=P(A)\cdot P(B')+P(B)\cdot P(A')$

$$=\frac{1}{2}\times\frac{2}{3}+\frac{1}{2}\times\frac{1}{3}$$

$$=\frac{1}{3}+\frac{1}{6}=\frac{1}{2}$$