Given that the events A and B are such that $P(A)=\frac{1}{2}, P(A\cap B)=\frac{3}{5}$ and $P(B)=p$. Find p if they are (i) mutually exclusive (ii) independent.

Solution:

Given, $P(A)=\frac{1}{2}, P(A\cap B)=\frac{3}{5}$ and $P(B)=p$

(i) When A and B are mutually exclusive, $A\cap B=\phi$

$\therefore P(A\cap B)=0$

Since, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow \frac{3}{5}=\frac{1}{2}+p-0$

$\Rightarrow p=\frac{3}{5}-\frac{1}{2}=\frac{1}{10}$

(ii) When A and B are independent, $P(A\cap B)=P(A)\times P(B)=\frac{1}{2}p$

Since, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow \frac{3}{5}=\frac{1}{2}+p-\frac{1}{2}p$

$\Rightarrow \frac{3}{5}=\frac{1}{2}+\frac{p}{2}$

$\Rightarrow \frac{p}{2}=\frac{3}{5}-\frac{1}{2}=\frac{1}{10}$

$\Rightarrow \frac{p}{2}=\frac{2}{10}=\frac{1}{5}$