For the three events A, B, and C, P (exactly one of the events A or B occurs) = P (exactly one of the two events B or C occurs) = P(exactly one of the events C or A occurs) = p and P (all the three events occur simultaneously) = p^2, where $0 < p < 1/2$. Then the probability of at least one of the three events A, B and C occurring is ***(1996 - 2 Marks)***

(a) $\frac{3p+2p^2}{2}$

(b) $\frac{p+3p^2}{4}$

(c) $\frac{p+3p^2}{2}$

(d) $\frac{3p+2p^2}{4}$

We know that P (exactly one of A or B occurs)
$= P(A) + P(B) - 2P(A \cap B)$.

Therefore, $P(A) + P(B) - 2P(A \cap B) = p$ ... (1)

Similarly, $P(B) + P(C) - 2P(B \cap C) = p$ ... (2)

and $P(C) + P(A) - 2P(C \cap A) = p$ ...(3)

Adding (1), (2) and (3) we get

$$2[P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A)] = 3p$$

$$\Rightarrow \quad P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) = 3p/2 \qquad \dots (4)$$

We are also given that,

$$P(A \cap B \cap C) = p^2 \qquad \dots (5)$$

Now, P (at least one of A, B and C)

$$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$= \frac{3p}{2} + p^2 \quad \text{[using (4) and (5)]} = \frac{3p + 2p^2}{2}$$