1 Prove that

(i) $P(A) = P(A \cap B) + P(A \cap \overline{B})$

(ii) $P(A \cup B) = P(A \cap B) + P(A \cap \overline{B}) + P(\overline{A} \cap B)$

i) $\because \quad P(A) = P(A \cap B) + P(A \cap \overline{B})$

$$\begin{aligned}
\therefore \quad \text{RHS} &= P(A \cap B) + P(A \cap \overline{B}) \\
&= P(A) \cdot P(B) + P(A) \cdot P(\overline{B}) \\
&= P(A)[P(B) + P(\overline{B})] \\
&= P(A)[P(B) + 1 - P(B)] && [\because P(\overline{B}) = 1 - P(B)] \\
&= P(A) = \text{LHS}
\end{aligned}$$

Hence proved.

i) $\because \quad P(A \cup B) = P(A \cap B) + P(A \cap \overline{B}) + P(\overline{A} \cap B)$

$$\begin{aligned}
\therefore \quad \text{RHS} &= P(A) \cdot P(B) + P(A) \cdot P(\overline{B}) + P(\overline{A}) \cdot P(B) \\
&= P(A) \cdot P(B) + P(A) \cdot [1 - P(B)] + [1 - P(A)]\, P(B) \\
&= P(A) \cdot P(B) + P(A) - P(A) \cdot P(B) + P(B) - P(A) \cdot P(B) \\
&= P(A) + P(B) - P(A) \cdot P(B) \\
&= P(A) + P(B) - P(A \cap B) \\
&= P(A \cup B) = \text{LHS}
\end{aligned}$$

Hence proved.