Let E_1 and E_2 be two independent events such that $P(E_1) = P_1$ and $P(E_2) = P_2$. Describe in words of the events whose probabilities are

(i) $P_1 P_2$ (ii) $(1 - P_1)P_2$

(iii) $1 - (1 - P_1)(1 - P_2)$ (iv) $P_1 + P_2 - 2P_1P_2$

$$P(E_1) = P_1 \text{ and } P(E_2) = P_2$$

i) $P_1P_2 \Rightarrow P(E_1) \cdot P(E_2) = P(E_1 \cap E_2)$

So, E_1 and E_2 occur.

ii) $(1 - P_1) P_2 = P(E_1)' \cdot P(E_2) = P(E_1' \cap E_2)$

So, E_1 does not occur but E_2 occurs.

ii) $1 - (1 - P_1)(1 - P_2) = 1 - P(E_1)'P(E_2)' = 1 - P(E_1' \cap E_2')$

$$= 1 - [1 - P(E_1 \cup E_2)] = P(E_1 \cup E_2)$$

So, either E_1 or E_2 or both E_1 and E_2 occurs.

v) $P_1 + P_2 - 2P_1P_2 = P(E_1) + P(E_2) - 2P(E_1) \cdot P(E_2)$

$$= P(E_1) + P(E_2) - 2P(E_1 \cap E_2)$$

$$= P(E_1 \cup E_2) - P(E_1 \cap E_2)$$

So, either E_1 or E_2 occurs but not both.