

4 A bag contains 5 red marbles and 3 black marbles. Three marbles are drawn one by one without replacement. What is the probability that at least one of the three marbles drawn be black, if the first marble is red?

Let $R = \{5 \text{ red marbles}\}$ and $B = \{3 \text{ black marbles}\}$

For at least one of the three marbles drawn be black, if the first marble is red, then the following three conditions will be followed

- (i) Second ball is black and third is red (E_1).
- (ii) Second ball is black and third is also black (E_2).
- (iii) Second ball is red and third is black (E_3).

$$\therefore P(E_1) = P(R_1) \cdot P(B_1 / R_1) \cdot P(R_2 / R_1 B_1) = \frac{5}{8} \cdot \frac{3}{7} \cdot \frac{4}{6} = \frac{60}{336} = \frac{5}{28}$$

$$P(E_2) = P(R_1) \cdot P(B_1 / R_1) \cdot P(B_2 / R_1 B_1) = \frac{5}{8} \cdot \frac{3}{7} \cdot \frac{2}{6} = \frac{30}{336} = \frac{5}{56}$$

$$\text{and } P(E_3) = P(R_1) \cdot P(R_2 / R_1) \cdot P(B_1 / R_1 R_2) = \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} = \frac{60}{336} = \frac{5}{28}$$

$$\begin{aligned} \therefore P(E) &= P(E_1) + P(E_2) + P(E_3) = \frac{5}{28} + \frac{5}{56} + \frac{5}{28} \\ &= \frac{10 + 5 + 10}{56} = \frac{25}{56} \end{aligned}$$