2 Refer to question 1 above. If the die were fair, determine whether or not the events A and B are independent.

Thinking Process

In a fair die, we have equally likely outcomes. So, with the given events A and B, we first find P(A), P(B) and P(A ∩ B) and then check whether they are dependent or independent.

Referring to the above solution, we have

$$A = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$$

$\Rightarrow \quad n(A) = 6$ and $n(S) = 6^2 = 36$ [where, S is sample space]

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

and $\quad B = \{(4, 6), (6, 4), (5, 5), (6, 5), (5, 6), (6, 6)\}$

$\Rightarrow \quad n(B) = 6$ and $n(S) = 6^2 = 36$

$$\therefore \quad P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Also, $\quad A \cap B = \{(5, 5), (6, 6)\}$

$\Rightarrow \quad n(A \cap B) = 2$ and $n(S) = 36$

$$\therefore \quad P(A \cap B) = \frac{2}{36} = \frac{1}{18}$$

Also, $\quad P(A) \cdot P(B) = \dfrac{1}{36}$

Thus, $\quad P(A \cap B) \neq P(A) \cdot P(B) \quad \left[\because \dfrac{1}{18} \neq \dfrac{1}{36}\right]$

So, we can say that both A and B are not independent events.

3 The probability that atleast one of the two events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.3, evaluate $P(\overline{A}) + P(\overline{B})$.

We know that, $A \cup B$ denotes the occurrence of atleast one of A and B and $A \cap B$ denotes the occurrence of both A and B, simultaneously.

Thus, $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.3$

Also, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow$ $0.6 = P(A) + P(B) - 0.3$

$\Rightarrow$ $P(A) + P(B) = 0.9$

$\Rightarrow$ $[1 - P(\overline{A})] + [1 - P(\overline{B})] = 0.9$ $\quad [\because P(A) = 1 - P(\overline{A})$ and $P(B) = 1 - P(\overline{B})]$

$\Rightarrow$ $P(\overline{A}) + P(\overline{B}) = 2 - 0.9 = 1.1$