2 Refer to question 1 above. If the die were fair, determine whether or not the events A and B are independent.

Thinking Process

In a fair die, we have equally likely outcomes. So, with the given events A and B, we first find P(A), P(B) and $P(A \cap B)$ and then check whether they are dependent or independent.

Referring to the above solution, we have

$$A = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$$

$$\Rightarrow n(A) = 6 \text{ and } n(S) = 6^{2} = 36 \qquad [where, S is sample space]$$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$
and
$$B = \{(4, 6), (6, 4), (5, 5), (6, 5), (5, 6), (6, 6)\}$$

$$\Rightarrow n(B) = 6 \text{ and } n(S) = 6^{2} = 36$$

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$
Also,
$$A \cap B = \{(5, 5), (6, 6)\}$$

$$\Rightarrow n(A \cap B) = 2 \text{ and } n(S) = 36$$

$$\therefore P(A \cap B) = \frac{2}{36} = \frac{1}{18}$$
Also,
$$P(A) \cdot P(B) = \frac{1}{36}$$
Thus,
$$P(A \cap B) \neq P(A) \cdot P(B)$$

$$\left[\because \frac{1}{18} \neq \frac{1}{36}\right]$$

So, we can say that both A and B are not independent events.