let $x_1, x_2, \ldots, x_{18}$ be eighteen observation such that $\sum_{i=1}^{18}(x_i-\alpha) = 36$ and $\sum_{i=1}^{18}(x_i-\beta)^2 = 90$, where α and β are distinct real numbers, if the standard deviation of these observations is 1, then value of $|\alpha-\beta|$ is ______

Ans: Given $\sum_{i=1}^{18}(x_i-\alpha) = 36$

$\Rightarrow \Sigma x_i - 18\alpha = 36$

$\Rightarrow \Sigma x_i - 18(\alpha+2) = 0$ — (1)

Also, $\sum_{i=1}^{18}(x_i-\beta)^2 = 90$

$\Rightarrow \Sigma x_i^2 + 18\beta^2 - 2\beta \Sigma x_i = 90$

$\Rightarrow \Sigma x_i^2 + 18\beta^2 + 2\beta \times 18(\alpha+2) = 90$

(using equation 1)

$\Rightarrow \Sigma x_i^2 = 90 - 18\beta^2 + 36\beta(\alpha+2)$

$\sigma^2 = 1 \Rightarrow \frac{1}{18}\Sigma x_i^2 - \left(\frac{\Sigma x_i}{18}\right)^2 = 1$ ($\because \sigma = 1$, given)

$\Rightarrow \frac{1}{18}(90 - 18\beta^2 + 36\alpha\beta + 72\beta) - \left(\frac{18(\alpha+2)}{18}\right)^2 = 1$

$\Rightarrow 90 - 18\beta^2 + 36\alpha\beta + 72\beta - 18(\alpha+2)^2 = 18$

$\Rightarrow 5 - \beta^2 + 2\alpha\beta + 4\beta - (\alpha+2)^2 = 1$

$\Rightarrow 5 - \beta^2 + 2\alpha\beta + 4\beta - \alpha^2 - 4 - 4\alpha = 1$

$\Rightarrow \alpha^2 - \beta^2 + 2\alpha\beta + 4\beta - 4\alpha = 0$

$\Rightarrow (\alpha-\beta)(\alpha-\beta+4) = 0$

$\Rightarrow \alpha - \beta = -4 \quad \therefore |\alpha-\beta| = 4$ $(\alpha \neq \beta)$