

Consider the statistics of two sets of observations as follows :

	Size	Mean	Variance
Observation I	10	2	2
Observation II	n	3	1

If the variance of the combined set of these two observations is $\frac{17}{9}$, then the value of n is equal to _____

Ans: For observation I :

$$\frac{\sum x_i}{10} = 2 \Rightarrow \sum x_i = 20$$

$$\frac{\sum x_i^2}{10} - 2^2 = 2$$

$$\Rightarrow \sum x_i^2 = 60$$

For observation II :

$$\frac{\sum y_i}{n} = 3 \Rightarrow \sum y_i = 3n$$

$$\frac{\sum y_i^2}{n} - 3^2 = 1 \Rightarrow \sum y_i^2 = 10n$$

Now, combined variance,

$$\sigma^2 = \left[\frac{\sum (x_i^2 + y_i^2)}{10+n} \right] - \left[\frac{\sum (x_i + y_i)}{10+n} \right]^2$$

$$\Rightarrow \frac{17}{9} = \frac{60 + 10n}{10+n} - \left(\frac{20 + 3n}{10+n} \right)^2$$

$$\Rightarrow 17(n^2 + 20n + 100) = 9(n^2 + 40n + 200)$$

$$\Rightarrow 8n^2 - 20n - 100 = 0$$

$$\Rightarrow 2n^2 - 5n - 25 = 0$$

$$\Rightarrow n = 5$$