10) A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize (a) at least once (b) exactly once (c) at least twice.

Ans: Let X represent the number of winning prizes in 50 lotteries. The trials are Bernoulli trials.

Clearly, X has a binomial distribution with $n = 50$ and $p = \frac{1}{100}$

$q = 1 - p = 1 - \frac{1}{100} = \frac{99}{100}$

$\therefore P(X = x) = {}^{n}C_x p^x q^{n-x} = {}^{50}C_x \left(\frac{99}{100}\right)^{50-x} \left(\frac{1}{100}\right)^{x}$

(a) P(winning at least once) $= P(X \geq 1)$

$= 1 - P(X < 1)$

$= 1 - P(X = 0)$

$= 1 - {}^{50}C_0 \left(\frac{99}{100}\right)^{50} = 1 - \left(\frac{99}{100}\right)^{50}$

(b) P(winning exactly once) $= P(X = 1)$

$= {}^{50}C_1 \left(\frac{1}{100}\right)^{1} \left(\frac{99}{100}\right)^{49} = 50 \left(\frac{1}{100}\right) \left(\frac{99}{100}\right)^{49}$

$= \frac{1}{2} \left(\frac{99}{100}\right)^{49}$

(c) $P(\text{At least twice}) = P(X \geq 2)$

$= 1 - P(X < 2)$

$= 1 - P(X \leq 1)$

$= 1 - [P(X=0) + P(X=1)]$

$= 1 - P(X=0) - P(X=1)$

$= 1 - \left(\frac{99}{100}\right)^{50} - \frac{1}{2}\left(\frac{99}{100}\right)^{49}$

$= 1 - \left(\frac{99}{100}\right)^{49}\left[\frac{99}{100} + \frac{1}{2}\right]$

$= 1 - \left(\frac{99}{100}\right)^{49}\left(\frac{149}{100}\right)$