9) On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by gessing?

Ans: The repeated guessing of correct answers from multiple choice questions are Bernoulli trials. Let X represent the number of correct answers by guessing in the set of 5 multiple choice questions.

Probability of getting a correct answer is

$$p = \frac{1}{3} \cdot \quad q = 1 - \frac{1}{3} = \frac{2}{3}$$

Clearly, X has a binomial distribution with

$n = 5$ and $p = \frac{1}{3}$.

$$\therefore P(X = x) = {}^{n}C_{x} \, p^{x} q^{n-x} = {}^{5}C_{x} \left(\frac{1}{3}\right)^{x} \left(\frac{2}{3}\right)^{5-x}$$

P(guessing more than 4 correct answers)

$$= P(X \geq 4) = P(X = 4) + P(X = 5)$$

$$= {}^{5}C_{4} \left(\frac{1}{3}\right)^{4} \left(\frac{2}{3}\right) + {}^{5}C_{5} \left(\frac{1}{3}\right)^{5}$$

$$= 5 \cdot \frac{2}{3} \cdot \frac{1}{81} + 1 \cdot \frac{1}{243} = \frac{10}{243} + \frac{1}{243} = \frac{11}{243}$$