

8) Suppose  $X$  has a binomial distribution  $B(6, \frac{1}{2})$

Show that  $x=3$  is the most likely outcome.  
(Hint:  $P(x=3)$  is the maximum among all  $P(x_i)$ ,  $x_i = 0, 1, 2, 3, 4, 5, 6$ )

Ans:  $X$  is the random variable whose binomial distribution is  $B(6, \frac{1}{2})$

Therefore,  $n=6$ ,  $p=\frac{1}{2}$

$$\therefore q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

Then,  $P(X=x) = {}^n C_x p^x q^{n-x}$

$$= {}^6 C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{6-x}$$

$$= {}^6 C_x \left(\frac{1}{2}\right)^6$$

It can be seen that  $P(X=x)$  will be maximum, if  ${}^6 C_x$  will be maximum

$$\text{Then } {}^6 C_0 = {}^6 C_6 = \frac{6!}{0! 6!} = 1$$

$${}^6 C_1 = {}^6 C_5 = \frac{6!}{1! 5!} = 6$$

$${}^6 C_2 = {}^6 C_4 = \frac{6!}{2! 4!} = 15$$

$${}^6C_3 = \frac{6!}{3!3!} = 20$$

The value of  ${}^6C_3$  is maximum. Therefore, for  $x=3$ ,  $P(X=x)$  is maximum.

Thus,  $x=3$  is the most likely outcome.