

7) In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true'; if it falls tails, he answers 'false'. Find the probability that he answers at least 12 questions correctly.

Ans:- let  $x$  present the number of correctly answered questions out of 20 questions. The repeated tosses of coin are Bernoulli trials. Since "head" on a coin represents the true answer and "tail" represents the false answer, the correctly answered questions are Bernoulli trials.

$$\therefore p = \frac{1}{2}, \quad q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

$x$  has a binomial distribution with  $n=20$ ,  $p=\frac{1}{2}$

$$\therefore P(X=x) = {}^n C_x p^x q^{n-x}, \quad x=0, 1, 2, \dots, n$$

$$= {}^{20} C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{20-x}$$

$$= {}^{20} C_x \left(\frac{1}{2}\right)^{20}$$

$P(\text{at least 12 questions answered correctly})$

$$= P(X \geq 12) = P(X=x) = {}^n C_x p^x q^{n-x}, \quad x=0, 1, 2, \dots, n$$

$P(\text{at least 12 questions answered correctly})$

$$= P(X \geq 12) = P(X=12) + P(X=13) + \dots + P(X=20)$$

$$= {}^{20}C_{12} \left(\frac{1}{2}\right)^{20} + {}^{20}C_{13} \left(\frac{1}{2}\right)^{20} + \dots + {}^{20}C_{20} \left(\frac{1}{2}\right)^{20}$$

$$= \left(\frac{1}{2}\right)^{20} \left[ {}^{20}C_{12} + {}^{20}C_{13} + \dots + {}^{20}C_{20} \right]$$