

5) The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05, what is the probability that out of 5 such bulbs i) none ii) not more than one iii) more than one will fuse after 150 days of use.

Ans: Let X represent the number of bulbs that will fuse after 150 days of use in an experiment of 5 trials. The trials are Bernoulli trials.

It is given that, $p = 0.05$

$$\therefore q = 1 - p = 1 - 0.05 = 0.95$$

X has a binomial distribution with $n = 5$ and $p = 0.05$

$$\therefore P(X = x) = {}^n C_x p^x q^{n-x}, \text{ where}$$
$$x = 0, 1, \dots, n$$

$$= {}^5 C_x (0.05)^x (0.95)^{5-x}$$

$$\begin{aligned}
 \text{i) } P(\text{none}) &= P(X=0) \\
 &= \sum_{k=0}^0 (0.95)^5 (0.05)^0 \\
 &= (0.95)^5
 \end{aligned}$$

$$\begin{aligned}
 \text{ii) } P(\text{not more than one}) &= P(X \leq 1) = P(X=0) + P(X=1) \\
 &= \sum_{k=0}^0 (0.95)^5 \times (0.05)^0 \\
 &\quad + \sum_{k=1}^1 (0.95)^4 \times (0.05)^1 \\
 &= (0.95)^5 + \cancel{0.25} \\
 &\quad + 5(0.05)(0.95)^4 \\
 &= (0.95)^4 [0.95 + 0.25] \\
 &= (0.95)^4 \times 1.2
 \end{aligned}$$

$$\begin{aligned}
 \text{iii) } P(\text{more than 1}) &= P(X > 1) \\
 &= 1 - P(X \leq 1) \\
 &= 1 - P(\text{not more than 1}) \\
 &= 1 - (0.95)^4 \times 1.2
 \end{aligned}$$

$$\begin{aligned}
 \text{iv) } P(\text{at least one}) &= P(X \geq 1) \\
 &= 1 - P(X < 1) = 1 - P(X=0) \\
 &= 1 - \sum_{k=0}^0 (0.05)^0 (0.95)^5
 \end{aligned}$$

$$= \sum_{i=0}^5 (0.95)^5 (0.05)^0$$

$$= (0.95)^5$$

$$\text{ii) } P(\text{not more than one})$$

$$= P(X \leq 1) = P(X=0) + P(X=1)$$

$$= \sum_{i=0}^0 (0.95)^5 \times (0.05)^0$$

$$+ \sum_{i=1}^1 (0.95)^4 \times (0.05)^1$$

$$= (0.95)^5 + (0.25)$$

$$+ 5 (0.05) (0.95)^4$$

$$= (0.95)^4 [0.95 + 0.25]$$

$$= (0.95)^4 \times 1.2$$

$$\text{iii) } P(\text{more than 1}) = P(X > 1)$$

$$= 1 - P(X \leq 1)$$

$$= 1 - P(\text{not more than 1})$$

$$= 1 - (0.95)^4 \times 1.2$$

$$\text{iv) } P(\text{at least one}) = P(X \geq 1)$$

$$= 1 - P(X < 1) = 1 - P(X=0)$$

$$= 1 - \sum_{i=0}^0 (0.05)^0 (0.95)^5$$

$$= 1 - (0.95)^5$$