4) Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that (i) all five cards are spades?
(ii) only 3 cards are spades?
(iii) none is a spade?

Ans:- Let X represent the number of spades cards among the five cards drawn. Since the drawing of card is with replacement, the trials are Bernoulli trials.

In a well shuffled deck of 52 cards, there are 13 spade cards.

$\Rightarrow p = \frac{13}{52} = \frac{1}{4}, \; q = 1 - \frac{1}{4} = \frac{3}{4}$

X has a binomial distribution with $n = 5$ and $p = \frac{1}{4}$.

$P(X = x) = {}^{n}C_{x} \, p^{x} q^{n-x}$, where $x = 0, 1, \ldots, n$

$= {}^{5}C_{x} \left(\frac{1}{4}\right)^{x} \left(\frac{3}{4}\right)^{5-x}$

i) P(all five cards are spades)

$= P(X = 5)$

$= {}^{5}C_{5} \left(\frac{1}{4}\right)^{5} \cdot \left(\frac{3}{4}\right)^{0} = 1 \cdot \frac{1}{1024} = \frac{1}{1024}$

ii) P(only 3 cards are spades)

$= P(X = 3) = {}^{5}C_{3} \left(\frac{1}{4}\right)^{3} \left(\frac{3}{4}\right)^{2}$

$= 10 \cdot \frac{9}{16} \cdot \frac{1}{64} = \frac{45}{512}$

iii) P(none is a spade) $= P(X = 0)$

$= {}^{5}C_{0} \cdot \left(\frac{1}{4}\right)^{0} \cdot \left(\frac{3}{4}\right)^{5} = 1 \cdot \frac{243}{1024}$

$= \frac{243}{1024}$