

3) There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

Ans:- Let X denote the number of defective items in a sample of 10 items drawn successively. Since the drawing is done with replacement, the trials are Bernoulli trials.

$$\Rightarrow p = \frac{5}{100} = \frac{1}{20}$$

$$\therefore q = 1 - \frac{1}{20} = \frac{19}{20}$$

X has a binomial distribution with $n=10$ and $p = \frac{1}{20}$

$$\therefore P(X=x) = {}^n C_x p^x q^{n-x}, \text{ where } x=0, 1, 2, \dots, n$$
$$= {}^{10} C_x \left(\frac{1}{20}\right)^x \left(\frac{19}{20}\right)^{10-x}$$

$$P(\text{not more than 1 defective item})$$
$$= P(X \leq 1) = P(X=0) + P(X=1)$$
$$= {}^{10} C_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^{10} + {}^{10} C_1 \left(\frac{1}{20}\right)^1 \left(\frac{19}{20}\right)^9$$
$$= \left(\frac{19}{20}\right)^{10} + 10 \left(\frac{19}{20}\right)^9 \left(\frac{1}{20}\right)$$

$$= \left(\frac{19}{20} \right)^9 \left[\frac{19}{20} + \frac{10}{20} \right]$$

$$= \left(\frac{19}{20} \right)^9 \cdot \left(\frac{29}{20} \right)$$