

Binomial Distribution

→ A die is thrown 6 times. If 'getting an odd number' is a success, what is the probability of i) 5 successes? ii) at least 5 successes iii) at most 5 successes?

Ans:- The repeated tosses of a die are Bernoulli trials. Let X denote the number of successes of getting odd numbers in an experiment of 6 trials.

Probability of getting an odd number in a single throw of a die is, $p = \frac{3}{6} = \frac{1}{2}$

$$\therefore q = 1 - p = \frac{1}{2}$$

X has a binomial distribution.

$$\text{Therefore, } P(X = x) = {}^n C_x p^x q^{n-x},$$

where $x = 0, 1, 2, \dots, n$

$$= {}^6 C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{6-x}$$

$$= {}^6 C_x \left(\frac{1}{2}\right)^6$$

$$i) P(5 \text{ successes}) = P(X=5)$$

$$= {}^6 C_5 \left(\frac{1}{2}\right)^6$$

$$= \frac{6}{64}$$

$$= \frac{3}{32}$$

$$ii) P(\text{at least 5 successes}) = P(X \geq 5)$$

$$= P(X=5) + P(X=6)$$

$$= {}^6 C_5 \left(\frac{1}{2}\right)^6 + {}^6 C_6 \left(\frac{1}{2}\right)^6$$

$$= \frac{6}{64} + \frac{1}{64} = \frac{7}{64}$$

$$\text{iii) } P(\text{at most 5 successes}) = P(X \leq 5)$$

$$= 1 - P(X > 5)$$

$$= 1 - P(X = 6)$$

$$= 1 - {}^6C_6 \left(\frac{1}{2}\right)^6$$

$$= 1 - \frac{1}{64} = \frac{63}{64}$$