

78) The probability of a shooter hitting a target is $\frac{3}{4}$. How many minimum number of times must he/she fire so that the probability of hitting the target at least once is more than 0.99?

Ans: Let the shooter fire n times. Obviously, n fires are n Bernoulli trials. In each trial, p = probability of hitting the target

$$= \frac{3}{4}$$

q = probability of not hitting the target = $\frac{1}{4}$

$$\begin{aligned} \text{Then } P(X=x) &= {}^n C_x q^{n-x} p^x \\ &= {}^n C_x \left(\frac{1}{4}\right)^{n-x} \left(\frac{3}{4}\right)^x \\ &= {}^n C_x \frac{3^x}{4^n} \end{aligned}$$

Now, given that,

$P(\text{hitting the ~~target~~ target at least once}) > 0.99$

$$\Rightarrow P(X \geq 1) > 0.99$$

Therefore $1 - P(X=0) > 0.99$

$$\Rightarrow 1 - {}^n C_0 \frac{1}{4^n} > 0.99$$

$$\Rightarrow {}^n C_0 \frac{1}{4^n} < 0.01$$

$$\Rightarrow \frac{1}{4^n} < 0.01$$

$$\Rightarrow 4^n > \frac{1}{0.01} = 100 \quad \text{--- (i)}$$

The minimum value of n to satisfy the inequality (i) is 4.

Thus, the shooter must fire 4 times.