

57 Coloured balls are distributed in four boxes as shown in the following table:

Box	Colour			
	Black	White	Red	Blue
I	3	4	5	6
II	2	2	2	2
III	1	2	3	1
IV	4	3	1	5

A box is selected at random and then a ball is randomly drawn from the selected box. The colour of the ball is black, what is the probability that ball drawn from the box III?

Ans: let A, E_1, E_2, E_3 and E_4 be the events as defined below:

A : a black ball is selected

E_1 : box I is selected

E_2 : box II is selected

E_3 : box III is selected

E_4 : box IV is selected

Since the boxes are chosen at random

$$\text{Therefore } P(E_1) = P(E_2) = P(E_3) = P(E_4) \\ = \frac{1}{4}$$

$$\text{Also } P(A|E_1) = \frac{3}{18}, P(A|E_2) = \frac{2}{8},$$

$$P(A|E_3) = \frac{1}{7}, P(A|E_4) = \frac{4}{13}$$

$P(\text{box III is selected, given that the drawn ball is black})$

$$= P(E_3|A)$$

By Bayes' theorem,

$$P(E_3|A) = \frac{P(E_3) \cdot P(A|E_3)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2) + P(E_4) \cdot P(A|E_4)}$$

$$= \frac{\frac{1}{4} \times \frac{1}{7}}$$

$$\frac{\frac{1}{4} \times \frac{3}{18} + \frac{1}{4} \times \frac{1}{7} + \frac{1}{4} \times \frac{1}{7} + \frac{1}{4} \times \frac{4}{13}}$$

$$= 0.165$$