

2) Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of kings.

⇒ let  $X$  denote the number of kings in a draw of two cards.  $X$  is a random variable which can assume the values 0, 1 or 2.

Now  $P(X=0) = P(\text{no king})$ .

$$\begin{aligned} P(X=0) &= \frac{{}^{48}C_2}{{}^{52}C_2} \\ &= \frac{48!}{2!(48-2)!} \\ &= \frac{52!}{2!(52-2)!} \\ &= \frac{48 \times 47}{52 \times 51} = \frac{188}{221} \end{aligned}$$

$P(X=1) = P(\text{one king and non-king})$

$$\begin{aligned} &= \frac{{}^4C_1 \cdot {}^{48}C_1}{{}^{52}C_2} = \frac{4 \times 48 \times 2}{52 \times 51} = \frac{32}{221} \end{aligned}$$

$$\begin{aligned} P(X=2) &= P(\text{two kings}) = \frac{{}^4C_2}{{}^{52}C_2} = \frac{4 \times 3}{52 \times 51} \\ &= \frac{1}{221} \end{aligned}$$

Thus, the probability distribution of  $X$  is

|        |                   |                  |                 |
|--------|-------------------|------------------|-----------------|
| $X$    | 0                 | 1                | 2               |
| $P(X)$ | $\frac{188}{221}$ | $\frac{32}{221}$ | $\frac{1}{221}$ |

Mean of  $X = E(X) = \sum_{i=1}^n x_i p(x_i)$

$$= 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + 2 \times \frac{1}{221}$$
$$= \frac{34}{221}$$

$$E(X^2) = \sum_{i=1}^n x_i^2 p(x_i)$$

$$= 0^2 \times \frac{188}{221} + 1^2 \times \frac{32}{221} + 2^2 \times \frac{1}{221}$$
$$= \frac{36}{221}$$

$$\text{var}(X) = E(X^2) - [E(X)]^2$$

$$= \frac{36}{221} - \left(\frac{34}{221}\right)^2 = \frac{6800}{(221)^2}$$

$$\sigma_X = \sqrt{\text{var}(X)} = \frac{\sqrt{6800}}{221} = 0.37$$