

Find the variance of the number obtained on a throw of an unbiased die.

⇒ The sample space of the experiment

$$S = \{1, 2, 3, 4, 5, 6\}$$

Let X denote the number obtained on the throw. Then X is a random variable which can take values 1, 2, 3, 4, 5 or 6.

$$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$$

The probability distribution of X is

X	1	2	3	4	5	6
$P(X)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$E(X) = \sum_{i=1}^n x_i p(x_i)$$

$$= 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6} = \frac{21}{6}$$

$$E(X^2) = 1^2 \times \frac{1}{6} + 2^2 \times \frac{1}{6} + 3^2 \times \frac{1}{6} + 4^2 \times \frac{1}{6} + 5^2 \times \frac{1}{6} + 6^2 \times \frac{1}{6} = \frac{91}{6}$$

$$\text{var}(X) = E(X^2) - (E(X))^2$$

$$= \frac{91}{6} - \left(\frac{21}{6}\right)^2 = \frac{91}{6} - \frac{441}{36}$$

$$= \frac{35}{12}$$