

Question 6: If $f: \mathbb{R} \rightarrow \mathbb{R}$ satisfy $f(x + y) = f(x) + f(y)$, for all $x, y \in \mathbb{R}$ and $f(1) = 7$ then

$\sum_{r=1}^n f(r)$ is

(a) $7n(n + 1)$

(b) $7n/2$

(c) $7(n + 1)/2$

(d) $7n(n + 1)/2$

Answer: (d)

Solution:

$f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$

$\rightarrow f(1 + 1) = 2f(1) = 2(7),$

$\rightarrow f(2) = 2(7)$

Therefore, $f(3) = f(1) + f(2) = 7 + 2(7) = 3(7)$

$\Rightarrow f(x) = ax$

$\Rightarrow a(1) = 7$

$\Rightarrow a = 7$

$f(x) = 7x$

$\sum_{r=1}^n f(r) = 7(1+2r+\dots+n) = 7n(n + 1)/2$