

5. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x$, $x \neq 0$, and $S = \{x \in \mathbb{R} : f(x) = f(-x)\}$; then S :

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- (1) contains more than two elements. (2) is an empty set.
(3) contains exactly one element (4) contains exactly two elements

Soln.->

$$5. \quad f(x) + 2f(1/x) = 3x \quad \dots (1)$$

$$x \rightarrow \frac{1}{x} \Rightarrow f(1/x) + 2f(x) = 3/x \quad \dots (2)$$

$$f(x) + 2\left(\frac{3}{x} - 2f(x)\right) = 3x$$

$$\Rightarrow 3f(x) = \frac{6}{x} - 3x$$

$$\Rightarrow f(x) = \frac{2}{x} - x$$

$$\text{For } S \quad f(x) = f(-x)$$

$$\Rightarrow \frac{2}{x} - x = 0$$

$$\Rightarrow x = \pm\sqrt{2}$$