

14. Let $f: R \rightarrow R$ be the function defined by $f(x) = \frac{1}{2 - \cos x}$, $\forall x \in R$. Then, find the range of f .

Sol. We have $f: R \rightarrow R$, $f(x) = \frac{1}{2 - \cos x}$, $\forall x \in R$.

$$\text{Let } y = \frac{1}{2 - \cos x}$$

$$\Rightarrow 2y - y \cos x = 1$$

$$\Rightarrow \cos x = \frac{2y - 1}{y}$$

$$\Rightarrow \cos x = 2 - \frac{1}{y}$$

Now we know that $-1 \leq \cos x \leq 1$

$$\Rightarrow -1 \leq 2 - \frac{1}{y} \leq 1$$

$$\Rightarrow -3 \leq -\frac{1}{y} \leq -1$$

$$\Rightarrow 1 \leq \frac{1}{y} \leq 3$$

$$\Rightarrow \frac{1}{3} \leq y \leq 1$$

So, range is $\left[\frac{1}{3}, 1\right]$