Equivalence Relation

A relation R in a set A is said to be an equivalence relation if R is **reflexive, symmetric and transitive**

E.g.: Height of Boys R = {(a, a) : Height of a **is equal to** height of a }

Set of all triangles in plane with R relation in T given by R = {(T1, T2) : T1 is congruent to T2}.

Numerical: Show that the relation R in the set {1, 2, 3} given by R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)} is reflexive but neither symmetric nor transitive.

Solution:

Since Relation R has elements {(1, 1), (2, 2), (3, 3)}, so I is Reflexive

Relation R has (1, 2), but, it doesn't have (2,1), so it is not symmetric

Relation R has (1, 2) & (2, 3), but it doesn't have (1, 3), so it is not transitive

Numerical: Determine if relation is reflexive, symmetric and transitive: Relation R in the set A of human beings in a town at a particular time given by

- R = {(x, y) : x and y work at the same place}
- R = {(x, y) : x is exactly 7 cm taller than y}

Solution: Lets solve for R = {(x, y) : x and y work at the same place} first.

The relation will have values (x,x), (y,y) also, since x & x will work at same place. So it is reflexive

If x & y works at same place, then y & x will also work at same place.

This relation R will have values (x,y)(y,x), so it is Transitive too.

If x & y works at same place, also it y & z works at same place, it implies that x & z works at same place.

Thus relation R will have value (x,y), (y,z), (x,z), so it is transitive too.

Thus it is equivalence relation.

Let's take case 2: R = {(x, y) : x is exactly 7 cm taller than y}, that is $x-y=7$

$x-x=0$, not 7. Thus the relation will not have (x,x), so it is not reflexive

$x-y \neq y-z$, so if relation R will have (x,y), it will not have (y,x), so it is not symmetric.

If $x-y=7$, & $y-z=7$, then $x-z = 14$, not 7.

Thus if relation has (x,y) & (y,z) elements, it will not have (x,z), so it is not transitive.