JEE Main 2021 (Online) 1st September Evening Shift

MCQ (Single Correct Answer)

Let $J_{n,m} = \int\limits_0^{\frac{1}{2}} \frac{x^n}{x^m - 1} dx$, $\forall$ n > m and n, m $\in$ N. Consider a matrix $A = [a_{ij}]_{3\times 3}$ where $a_{ij} = \begin{cases} J_{6+i,3} - J_{i+3,3}, & i \le j \\ 0, & i > j \end{cases}$. Then $|adj A^{-1}|$ is :

(A) $(15)^2 \times 2^{42}$

(B) $(15)^2 \times 2^{34}$

(C) $(105)^2 \times 2^{38}$

(D) $(105)^2 \times 2^{36}$

Explanation

A = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$J_{6+i,3} - J_{i+3,3}; i \le j$

$= \int_0^{\frac{1}{2}} \frac{x^{6+i}}{x^3 - 1} - \int_0^{\frac{1}{2}} \frac{x^{i+3}}{x^3 - 1}$

$= \int_0^{1/2} \frac{x^{i+3}(x^3 - 1)}{x^3 - 1}$

$= \frac{x^{3+i+1}}{3+i+1} = \left(\frac{x^{4+i}}{4+i}\right)_0^{1/2}$

$\therefore a_{ij} = j_{6+i,3} - j_{i+3,3} = \frac{\left(\frac{1}{2}\right)^{4+i}}{4+i}$

$a_{11} = \frac{\left(\frac{1}{2}\right)^5}{5} = \frac{1}{5.2^5}$

$a_{12} = \frac{1}{5.2^5}$

$a_{13} = \frac{1}{5.2^5}$

$a_{22} = \frac{1}{6.2^6}$

$a_{23} = \frac{1}{6.2^6}$

$a_{33} = \frac{1}{7.2^7}$

$A = \begin{bmatrix} \frac{1}{5.2^5} & \frac{1}{5.2^5} & \frac{1}{5.2^5} \\ 0 & \frac{1}{6.2^6} & \frac{1}{6.2^6} \\ 0 & 0 & \frac{1}{7.2^7} \end{bmatrix}$

$|A| = \frac{1}{5.2^5}\left[\frac{1}{6.2^6} \times \frac{1}{7.2^7}\right]$

$|A| = \frac{1}{210.2^{18}}$

$|adj A^{-1}| = |A^{-1}|^{n-1} = |A^{-1}|^2 = \frac{1}{(|A|)^2}$

$= (210.2^{18})^2$

$= (105)^2 \times 2^{38}$