

1 JEE Main 2021 (Online) 20th July Evening Shift

Numerical

Let $A = \{a_{ij}\}$ be a 3×3 matrix,

$$\text{where } a_{ij} = \begin{cases} (-1)^{j-i} & \text{if } i < j, \\ 2 & \text{if } i = j, \\ (-1)^{i+j} & \text{if } i > j \end{cases}$$

then $\det(3\text{Adj}(2A^{-1}))$ is equal to _____.

Answer

Correct Answer is 108

Explanation

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$|A| = 4$$

$$\det(3\text{adj}(2A^{-1}))$$

$$= 3^3 |\text{adj}(2A^{-1})|$$

$$= 3^2 |2A^{-1}|^2$$

$$= 3^3 \cdot 2^2 |A^{-1}|^2 = 3^3 \cdot 2^2 \cdot \frac{1}{|A|^2} = 3^2 \cdot 2^2 \cdot \frac{1}{4^2} = 108$$