

3 JEE Main 2021 (Online) 17th March Morning Shift
Numerical

If $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$, then the value of $\det(A^4) + \det(A^{10} - (\text{Adj}(2A))^{10})$ is equal to
_____.

Answer

Correct Answer is 16

Explanation

$$A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$$

$$|A| = -2 \Rightarrow |A|^4 = 16$$

$$A^2 = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 8 & 9 \\ 0 & -1 \end{bmatrix}$$

$$\therefore A^{10} = \begin{bmatrix} 2^{10} & 2^{10} - 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1024 & 1023 \\ 0 & 1 \end{bmatrix}$$

$$2A = \begin{bmatrix} 4 & 6 \\ 0 & -2 \end{bmatrix}$$

$$\text{adj}(2A) = \begin{bmatrix} -2 & -6 \\ 0 & 4 \end{bmatrix}$$

$$\text{adj}(2A) = -2 \begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}$$

$$(\text{adj}(2A))^{10} = 2^{10} \begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}^{10}$$

$$= 2^{10} \begin{bmatrix} 1 & -(2^{10} - 1) \\ 0 & 2^{10} \end{bmatrix}$$

$$= 2^{10} \begin{bmatrix} 1 & -1023 \\ 0 & 1024 \end{bmatrix}$$

$$A^{10} - (\text{adj}(2A))^{10} = \begin{bmatrix} 0 & 2^{11} \times 1023 \\ 0 & 1 - (1024)^2 \end{bmatrix}$$

$$|A^{10} - \text{adj}(2A)^{10}| = 0$$

$$\therefore \det(A^4) + \det(A^{10} - (\text{Adj}(2A))^{10})$$

$$= 16 + 0 = 16$$