1 JEE Main 2021 (Online) 27th August Morning Shift

MCQ (Single Correct Answer)

If the matrix $A = \begin{pmatrix} 0 & 2 \\ K & -1 \end{pmatrix}$ satisfies $A(A^3 + 3I) = 2I$, then the value of K is :

A $\frac{1}{2}$

B $-\frac{1}{2}$

C -1

D 1

Explanation

Given matrix $A = \begin{bmatrix} 0 & 2 \\ k & -1 \end{bmatrix}$

$A^4 + 3IA = 2I$

$\Rightarrow A^4 = 2I - 3A$

Also characteristic equation of A is $|A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} 0-\lambda & 2 \\ k & -1-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda + \lambda^2 - 2k = 0$

$\Rightarrow A + A^2 = 2K.I$

$\Rightarrow A^2 = 2KI - A$

$\Rightarrow A^4 = 4K^2I + A^2 - 4AK$

Put $A^2 = 2KI - A$

and $A^4 = 2I - 3A$

$2I - 3A = 4K^2I + 2KI - A - 4AK$

$\Rightarrow I(2 - 2K - 4K^2) = A(2 - 4K)$

$\Rightarrow -2I(2K^2 + K - 1) = 2A(1 - 2K)$

$\Rightarrow -2I(2K - 1)(K + 1) = 2A(1 - 2K)$

$\Rightarrow (2K - 1)(2A) - 2I(2K - 1)(K + 1) = 0$

$\Rightarrow (2K - 1)[2A - 2I(K + 1)] = 0$

$\Rightarrow K = \frac{1}{2}$