Q3.

34. If A and B are invertible matrices, then which of the following is not correct?

    (a) adj. $A = |A| \cdot A^{-1}$.

    (b) $\det(A)^{-1} = [\det(A)]^{-1}$

    (c) $(AB)^{-1} = B^{-1}A^{-1}$

    (d) $(A + B)^{-1} = B^{-1} + A^{-1}$

Sol. (d) Given A and B are invertible matrices.

Now $(AB)B^{-1} A^{-1}$

$$= A(BB^{-1})A^{-1} = AIA^{-1} = (AI)A^{-1} = AA^{-1} = I$$

$$\Rightarrow \qquad (AB)^{-1} = B^{-1} A^{-1}$$

Also $A A^{-1} = I$

$$\Rightarrow \qquad |A A^{-1}| = |I|$$

$$\Rightarrow \qquad |A||A^{-1}| = 1$$

$$\Rightarrow \qquad |A^{-1}| = \frac{1}{|A|}$$

$$\Rightarrow \qquad \det(A)^{-1} = [\det (A)]^{-1}$$

Also we know that $A^{-1} = \dfrac{\text{adj. } A}{|A|}$

$$\Rightarrow \qquad \text{adj. } A = |A| \cdot A^{-1}$$

$$(A + B)^{-1} = \frac{1}{|A + B|} \text{adj.} (A + B)$$

But $\qquad B^{-1} + A^{-1} = \dfrac{1}{|B|}\text{adj. } B + \dfrac{1}{|A|}\text{adj. } A$

$$\Rightarrow \qquad (A + B)^{-1} \neq B^{-1} + A^{-1}$$