JEE Main 2021 (Online) 22th July Evening Shift

MCQ (Single Correct Answer)

Let the circle S : $36x^2 + 36y^2 - 108x + 120y + C = 0$ be such that it neither intersects nor touches the co-ordinate axes. If the point of intersection of the lines, $x - 2y = 4$ and $2x - y = 5$ lies inside the circle S, then :

A. $\frac{25}{9} < C < \frac{13}{3}$

B. $100 < C < 165$

C. $81 < C < 156$

D. $100 < C < 156$

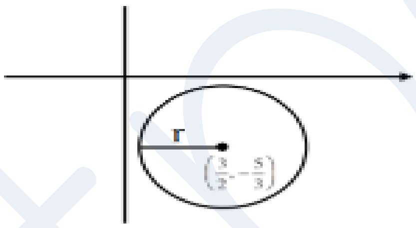
Explanation

S : $36x^2 + 36y^2 - 108x + 120y + C = 0$

$\Rightarrow x^2 + y^2 - 3x + \frac{10}{3}y + \frac{C}{36} = 0$

Centre $= (-g, -f) = \left(\frac{3}{2}, \frac{-10}{6}\right)$

radius $= r = \sqrt{\frac{9}{4} + \frac{100}{36} - \frac{C}{36}}$



Now,

$\Rightarrow r < \frac{3}{2}$

$\Rightarrow \frac{9}{4} + \frac{100}{36} - \frac{C}{36} < \frac{9}{4}$

$\Rightarrow C > 100$ ...... (1)

Now, point of intersection of $x - 2y = 4$ and $2x - y = 5$ is $(2, -1)$, which lies inside the circle S.

$\therefore S(2, -1) < 0$

$\Rightarrow (2)^2 + (-1)^2 - 3(2) + \frac{10}{3}(-1) + \frac{C}{36} < 0$

$\Rightarrow 4 + 1 - 6 - \frac{10}{3} + \frac{C}{36} < 0$

$C < 156$ ..... (2)

From (1) & (2)

$100 < C < 156$ Ans.